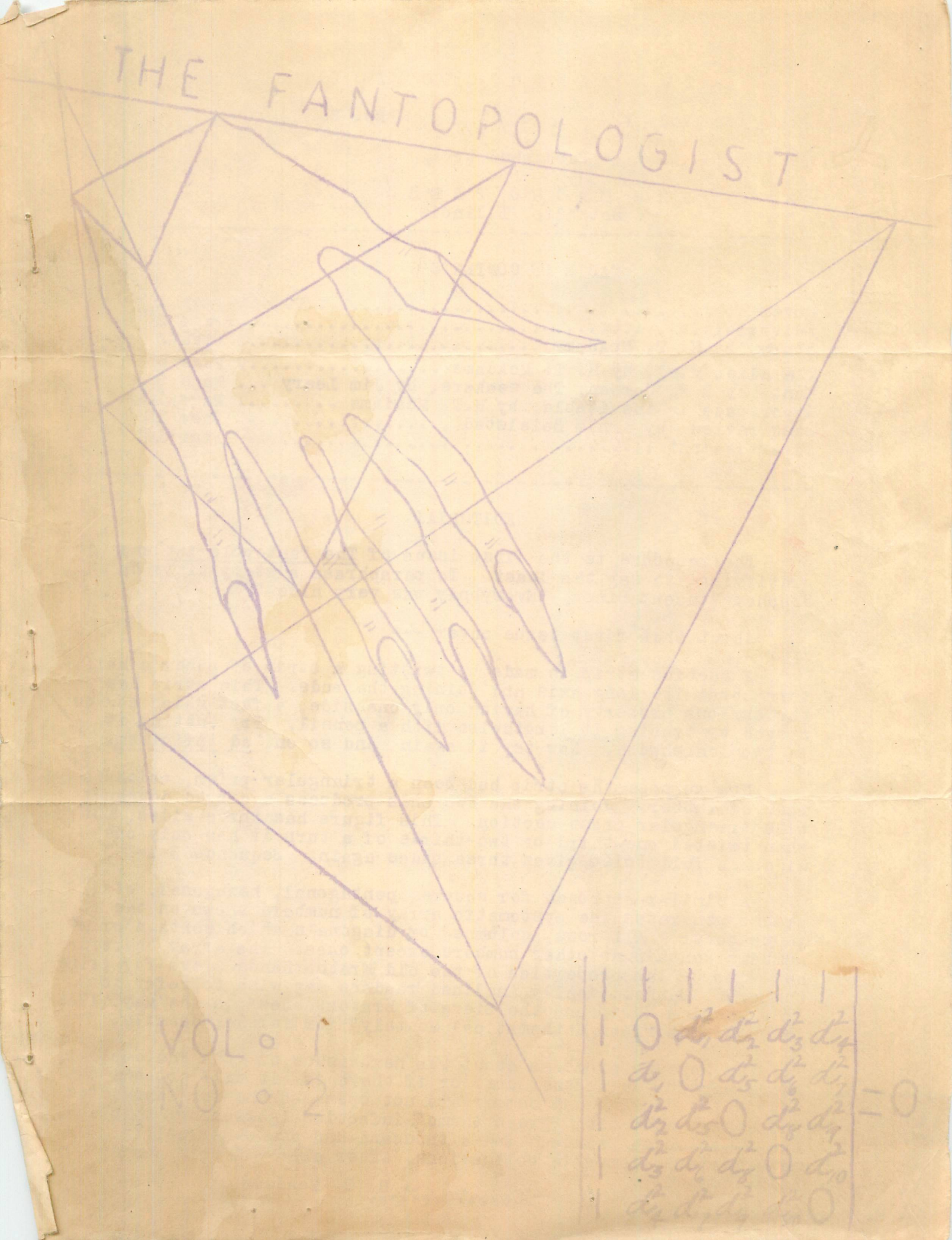
THE FANTOPOLOGIST



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NO. 2

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Vol. 1 No. 2

Published by

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Now suppose the strip had been a triangular prism, which is made of rubber. Joining the two ends produces a closed doughnut with triangular cross-section. This figure has three sides, but when twisted one-third or two-thirds of a turn it has only one side. A full twist gives three sides again. Sequence 3 1 1 3.

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-- H. T. McAdams

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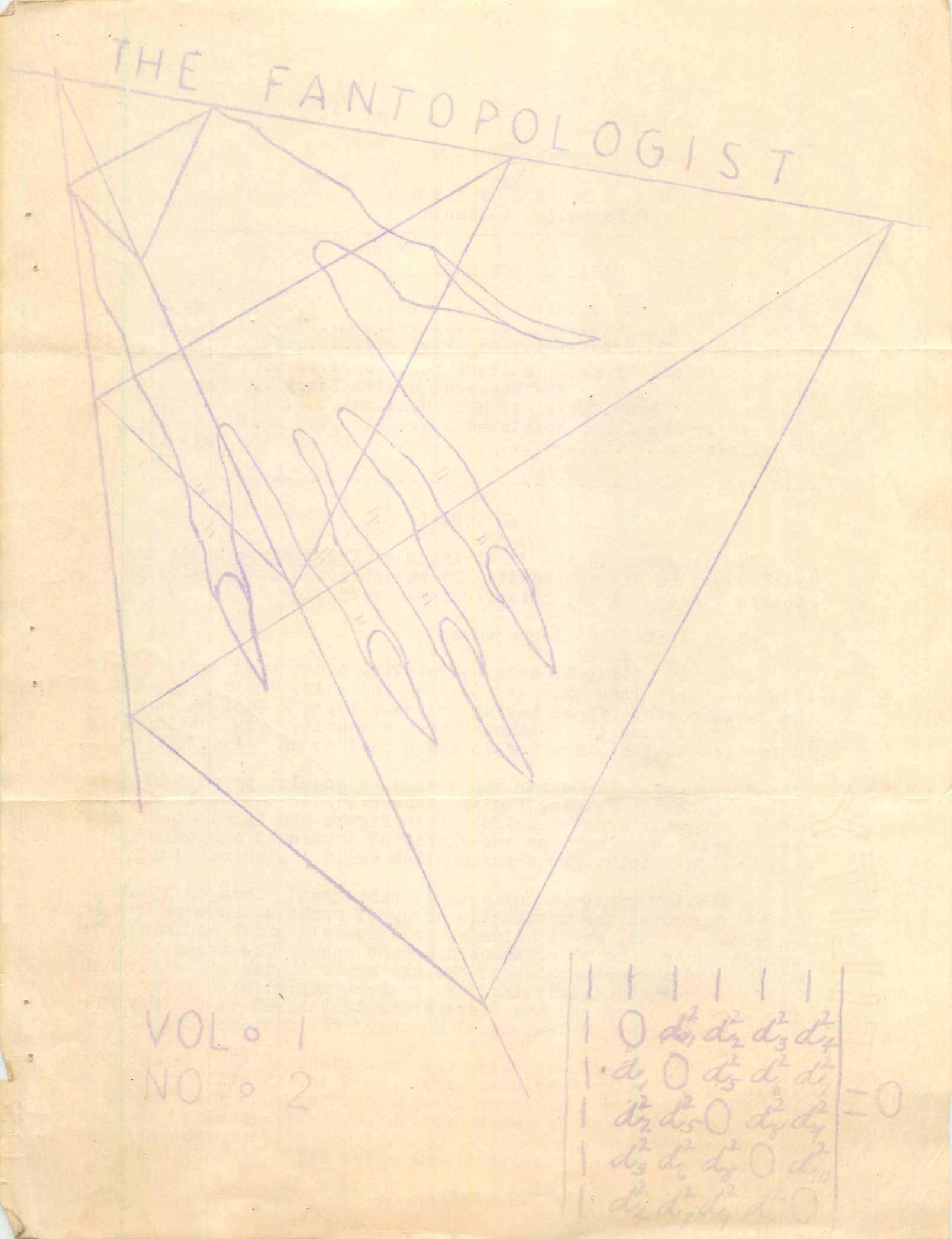

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WILDWOOD

by

H. T. MCADAMS

You are not always aware of the steel in your heart and in your nervous system and in your dealings with men. It is a fact which you ignore completely as you strive in frantic eagerness to close that Martian deal for Plexigon securities, or to swing that merger with Venusian Limited, or to corner more than your share of interstellar commerce. After all, that's your job. It leaves little time for poetic yearning.

But sometimes, as you watch a beetle car scuttle hurriedly down the rollane, or gaze covetously at a sleek steel rocket in its insistent leap skyward, you are suddenly conscious of the steel, and it bothers you a bit. You have an insane desire to live, to escape from this sterile, mechanical world and its space-warped sense of values, to fulfill a destiny that somehow beckons to you from somewhere in the depths of your soul.

Perhaps you have never felt it, that call! It comes in spring like a revelation of faith, in summer like a cool hand on a fevered brow. In autumn, it caresses your soul with melancholy gladness, and in winter it makes your peace with whatever gods may be.

And somehow you must answer that call, for it comes from within you, and from far-away places. Somewhere away from the space-port and the rollane, and the thousand-and-one other symbols of transient things, you have a rendezvous with hills, and streams, and skies.

That pilgrimage is an endless adventure. As you twist your tortuous way among blackberry briars and hedge, or through a grove of wild crab-apples, angry talons take their toll on face and hands and ankles. Grapevines, honeysuckle, and woodbine make a potential gallows of every tree, and poison ivy occasionally raises its trefoil threat in jealous competition.

As you descend a vertical hillside, your pony's hind legs suddenly seem to be severed at their joints as he slips on a loose stone and sends it clattering into the valley below. But he merely grunts annoyance at the troublesome pebble, and you hear his lips pop as he misses a bit of mullein which he would politely refuse at any other time.

Of course the pony is not necessary, not for transportation at least. An anti-grav belt would be far more efficient. But, after all, you can not confide in an anti-grav belt. With Billy, it is different. You have something in common. You are both slaves to the same purpose. Perhaps, you remind yourself, that is why the Martian vlaeth-bear, lovable creature that he is, has not been able to replace the dog in Man's affection, not even in a thousand years.

Safe at the bottom, you survey an expanse of lush bluegrass which is terminated only by more hills beyond. For a moment you seem about to be ravaging something sacred, but once you go swishing through the stuff, nearly up to Billy's knees, you forget your sin. Your path is swallowed up behind you, and no one will know your folly.

Wildwood (cont. from page 3.)

Time and distance are of little concern in this boundless universe. Hours or eons pass as you ride the strange waves of nothingness toward the western sun. But suddenly a frog plops into something liquid somewhere, and Billy asks for more rain as he drops his head, and, standing front feet in the water, luxuriously freshes himself beneath a large elm overhanging the stream.

The drama of life which unfolds before you is a far cry from the life of steel which you know. And yet, there is a certain similarity which you would rather not admit.

Tiny "craw-daddies," caught in a ruthless attack on an earthworm, scurry away like so many scared children caught stealing jam. And, in spite of yourself, you see the space-port and the impatient rockets, and the Plaxigon Exchange.

Little crawling things like sowbugs under old boards seem to symbolize the level of Man's cultural development.

And mud-puppies, those black little salamanders with squat little legs and fat little jaws, could well be real puppies after all, if they were not so reminiscent of Martian usurers.

As you lie in the shade of that great overhanging elm, you look above and below, but can perceive no essential difference. You are among the clouds either way. The vast blue above recedes and expands until it seems to be so all about you that you must find a bush, or a tree, or something to tie down to.

Perhaps you choose Billy for your anchor as he munches the luscious bluegrass, and as you watch those little pits above his eyes alternately bulge and sink. Or it may be a red lizard, six feet above you on the perpendicular hillside, staring at you from his snappy, bewildered eyes for a moment before he disappears. Again, it may be the first crow returning to the rookery, and you may break the spell long enough to mimic his expressive night cry, just to see if you can "bring him over."

Finally, as the shadows lengthen, the little stars peep out one by one, and the moon fulfills a long belated prophecy. The stream before you becomes a dancing flux of molten gold set with tiny, dancing jewels. A weird glow steals down the hillside, playing with shadows among the brambles and thickets, silhouetting many things, revealing few.

You rise to go now. You walk toward Billy and he looks at you, blinking his soulful eyes in the moonlight, the steam from his sides rising visibly into the night air. And suddenly you realize, for the first time perhaps, what Shakespeare meant about parting and sweet sorrow. For long before you wished it, you are on the roll ane, the symbol of transient things.

But perhaps it is I, after all, not you, who do these things. Perhaps you are not conscious of the steel in your heart and in your nervous system. After all, why should you be?

For you are human, while I am only a robot.

THE END

A FEW COPIES OF FANTOPOLOGIST #1 STILL AVAILABLE. 10¢ PER COPY.

THE ALIEN MIND

by

H. T. McAdams

In its endless attempt to escape from a humdrum terran existence, science fiction has progressed avidly from the extra-planetary to the extra-stellar to the extra-galactic frames of reference. Yet, in spite of the "alien-ness" of its settings, its characters all too often might have lived in Centerville, U.S.A., judging from the strictly human flavor of their emotions and ways of thinking.

Space, too, has remained pretty much its time-honored self as something through which rocket ships must pass in order to get from here to there. Though science fiction has created new planets, new solar systems, and new universes, it has pulled up sharply at the point at which it might have taken its boldest step -- namely, into the realm of the extra-spatial.

The concurrence of these two limitations upon the most imaginative of all literary outlets is too striking to be coincidental. In the hesitancy of science fiction to strike out boldly into these areas of the unknown, there seems to be a tacit admission that the mind has reached the end of its tether. Faced with the task of conceiving the inconceivable and of feeling that which is not subject to feeling, the human mind finds itself in the dilemma of being ... not human!

Science fiction needs not hide its chagrin at being in the position of the snake which attempted to swallow itself; it needs only to know that the feat can be accomplished. There is, in mathematics, a figure known as the Klein bottle, which has neither inside nor outside in the usual sense. This bottle may be thought of, in our imagination at least, as the snake who made good. In like manner it may be within the province of the human mind to conceive of a way of thinking alien to itself, and to reject the term "inconceivable" altogether. For in the very act of calling a thing "inconceivable" we have conceived of it already.

When asked how he happened to discover the principle of relativity, Einstein is said to have remarked that he merely challenged a postulate. Such challenging of accepted ideas is the science fiction writer's "open sesame" to an alien world. By rejecting the known he may acquire an intimate knowledge of the unknown.

Everyone is familiar with the individual who insists upon giving directions to a certain house by an exhaustive description of the one next to it, that description being followed by the inevitable "not that one, the next one." In like manner, the "inconceivable" may be disposed of by describing not so much what it is, as what it is not. Human logic is a logic of opposites, and it is this principle of negation, or contradiction, which affords at least an intuitive grasp of ideas too alien to be put explicitly into words.

The failure of science fiction to provide a new concept of space, and at the same time to develop a truly "alien" way of thinking, lies in the fact that spatial relationships are tantamount to any logical discussion of nature. In other words, thought itself is peculiarly spatial. In order to understand this connection, however, we must consider Space in its Mathematical rather than in its physical sense.

THE ALIEN MIND (cont. from p. 5)

The physical concept of space is inseparable from the concept of matter. The mathematical concept, on the other hand, is refreshingly devoid of all such irrevelant fictions. The mathematician begins with an aggregate of undefined elements and proceeds to develop from one or more simple postulates, a certain structure of those elements. This structure he calls a "space", and there are as many "spaces" as there are generating aggregates and postulates.

The mathematician is not in the least concerned with whether or not a given space so generated conforms with anything in his sensual experience; he is concerned only with whether it is logically consistent. Needless to say, by far the greater number of such spaces are contradictions of Nature as we ordinarily think of it. Obviously, then, the science fiction writer may look to the mathematician and be wise.

In any consideration of spatial concepts, number necessarily assumes an important role. In the terran system, for example, the number two holds, in many ways, a unique position. By virtue of a principle known as duality, mathematical concepts often occur in pairs, and theorems concerning one are applicable to the other by the simple expedient of interchanging the terms. For example, two points determine a line, and two lines determine a point. In other words, the terms "point" and "line" are duals which may be interchanged without affecting the truth of the statement.

Just how deeply this principle has its roots in our culture is difficult to ascertain. Dualism, in its philosophical aspects, has been pondered for centuries, and its relation to the mathematical principle is implicit, if not explicit. In modern physics the Uncertainty Principle smacks of a similar tincture, and a different statistical treatment is required for systems containing an even number than for those containing an odd number of fundamental particles.

Human logic, too, it must be remembered, is peculiarly two-valued, and remains so in spite of the innovations of non-Aristotelian logic. Modern multi-valued systems have found it necessary to introduce a "meta-language", which obeys the same old "true-false" restriction, regardless of the nature of the "object language."

A fictional escape from the "tyranny of two" therefore constitutes a real challenge to the science fiction writer. How can it be done?

A number of dualisms in our culture have developed with a greater emphasis upon one member of the dual than upon the other. By indulging "the other half" of such duals, in much the same manner that we indulge a left-handed child, we may re-locate the emphasis in such a way as to produce substantially "alien" cultures.

Numbers themselves, for example, may be classified as real or imaginary, rational or irrational, algebraic or transcendental, finite or transfinite. The terms themselves, in their metaphorical implications, are suggestive enough, but the ideas beneath these semantic garments have all the provocativeness of a burlesque queen. In alien systems of mathematics the imaginaries, the irrationals, the transcendentals, and the transfinites may be as familiar and manageable as 1, 2, 3.

In the field of geometry, to take a second familiar example, a certain dualism is implied between metric and topological relationships. Metric geometry is the geometry of measurement, and is concerned with such problems as the length of lines and the sizes of angles. Topology, on the other hand is the geometry of position, and is concerned only with those properties of geometrical figures which are independent of measurement.

A circle drawn on a sheet of rubber loses its metric properties when the sheet is stretched, but retains its topological properties. Although the circumference of the circle is changed the order or arrangement of the points on the distorted curve is not. In addition, the curve will continue to divide its plane into two parts, an inside and an outside, regardless of the extent to which it is stretched.

Topological relationships are not as trivial as the above example may imply, and abound all about us, from the design of a three-way electric switch to the chemical phase rule. Yet terran science, for the most part, has been based upon measurement, a la Cartesian analytical geometry, rather than upon topology, and has treated with only casual annoyance the topological problems which have been thrust upon it by an insistent universe.

How different might our culture have been had the situation been reversed! Such terms as "measurement" and "distance", if not completely unintelligible, might be understood only by the abstract mathematicians. Qualitative, or topological, relationships might be emphasized at the expense of quantitative, or metric relationships, and "qualitum physics" might well replace quantum physics.

A second means of terminating the dualistic dynasty is to reject it completely. What would a world be like if it had only a single-valued truth system? Why must there be a true and a false for every situation? Why, indeed, must there be a "why?" In a world of such logic, if we may call it that, there would be no cause and effect relationships: events would be substantially "uncaused." This is getting close to the alien goal of the "inconceivable", but is perhaps not too bold in view of certain recent revisions in the whole subject of causation.

It is obvious that the challenging of such philosophical concepts is concerned with extremely primitive ideas. These ideas strike deep into the roots of human nature and very often have their origin somewhere in the dawn of civilization.

The fact that plants reproduce from seed is considered the greatest of the "seven wonders" of discovery and invention. To most of us it has never occurred that this germ of agricultural economy needed to be discovered. Yet this is only one of a number of similar instances in our racial heritage.

When, for example, did Man learn to distinguish between himself and his environment? The first man to have felt this consciousness of self in a sense discovered Life.

There is, however, a sequel to this discovery, which is no less important. How long may Man have lived, noting generation after generation rise and fall, before associating this fact with the inevitability of death? How different would his world be if he had not made this discovery? For with it came Man's recognition of the moral values of life, as well as his mad craving for immortality.

What of the man who discovered Death? His story might well be told!

Aesthetics, no less than logic, is subject to a spatial structure if we bear in mind that "space" is to be interpreted in its strictly mathematical sense. This fact is implied by the importance of rhythm and symmetry in human emotions, as well as by the definite relationships which prevail among the various senses.

Non-living matter is, for the most part, crystalline. By definition the word "crystalline" implies an orderly and periodic arrangement of atoms, and a relatively high degree of symmetry. Non-living matter, on the other hand, seems to be "crystalline" by a somewhat different definition. Its atoms are arranged in an orderly way but in a way which is strictly non-periodic. This contrast between Man and his environment may well be the basis of his aesthetic sense.

Let us suppose now, in keeping with our spirit of contradiction, that "alien" races exist who are physically periodic but whose environment is non-periodic. What strange emotions would a Terran experience upon being exposed to such an aperiodic culture? Apartment house tenants have been known to lose an entire night's sleep waiting for a second shoe to drop, and in at least one instance a famous musician attempted suicide in an effort to escape from the mental persistence of a well-tuned "A".

An abnormality in the relationships among the various senses is familiar to psychologists under the name of synaesthesia. Sights, sounds, and smells become confused in such a way as to produce "shrill" colors, "loud" odors, "colorful" language, and musical "taste." The realization that such expressions are something more than metaphorical makes the challenging of the terran structure of sensation somewhat more reasonable. In addition, as Voltaire suggests in his "Micromegas", it is not unreasonable to believe that "alien" beings may possess senses entirely unknown to the Terran.

Some may object that an approach to "alien-ness" via the contradiction of "reality" is "nonsense." Not only are there destructive semantic arguments against such a circular statement, but powerful psychological ones as well. For it is only the conscious mind which insists on sanity. Dreams, for example, those bits of driftwood from the sea of subconsciousness, never seem to "make sense."

Why should the human mind deliberately suppress part of the knowledge which it receives through the senses? Why should such a chasm exist between the conscious and the subconscious? Apparently, if we are to believe in evolution, this adaptation developed because it had survival value; its presence in some way was a boon to Man's existence.

In view of the welter of confused sense impressions which constantly assail his consciousness, Man may have a very real need for a "Censor" to sort the grain from the chaff. The more deeply science delves into the mysteries of our existence, the more inconsistencies it unearths. In fact, the principle of anomaly has been proposed as a universal law no less rigid than the law of gravitation. By suppressing a certain amount of knowledge, therefore, the mind may be doing its noble best to save itself from frustration. In a more material way, the Censor may be protecting Man from self-extermination by depriving him of intimate facts which might be too dangerous for him to know.

THE ALIEN MIND (cont. from p. 8)

A being lacking in such subconscious inhibitions would be an alien being indeed! What demoniacal machines of science might such an alien produce? What irrational systems of logic would control his behavior? And, in view of the subconscious interpretations of the surrealists, what wild orgies of artistic creation would grace his art museums, his libraries, and his concert halls?

The science fiction writer, in his own way, attempts to solve these problems. In his effort to create an alien being, he becomes, in a sense an alien himself. For a while at least, he is the irrational and the transcendental, the mentally "inconceivable", the topologically different. Though not necessarily a Hamlet, he very literally has a "method in his madness."

In order to abbreviate what might otherwise be an exhaustive description, however, the science fiction writer's own pet device of negation is perhaps most effective. For, whatever else may be said of him, the science fiction writer who aspires to alien status is not a "yes-"yes-man."

He says "No" to everything.

****

KANT (A SONNET) by R. Flavie Carson

God is a logical necessity
If we are to have any wholesome view
Of all the universe, and to eschew
The case against all rationality;-
Such worlds as we have knowledge of are planned
Beyond all scope of mankind's intellect,
This earth of ours although a tiny speck,
Is still held vital in the Master's hand.

Thus reasoned Kant and Kant was one to know--
A pedant of the philosophic school,
He knew astronomy and logic's rule
And how men's minds sought to expand and grow;
Yes, Kant was one who surely thought things through,
But just how much of Kant did Kant think true?

****

THE SEEKERS by Jim Leary

From far out of the night come fingers
Black fingers probing for my soul.
I recoil, and am safe ...
But if I should rest
If sleep should cross the pathways of my mind
The fingers would search, and search, and find.

I hear their rasping in some far space;
Fear rules my stomach
Fear -- for I grow tired
And the hands of darkness come closer.

****

HYPERSPACE IS INEVITABLE!

(A discussion of abstract metrics)

by H. T. McAdams

Because it serves as such a ready means of escape from the "Here and Now", space of higher dimensions exerts a powerful influence upon the destiny of science-fiction. From certain valid mathematical analogies, however, such as H. G. Wells employed in "The Time Machine", the term "dimension" has become so semantically perverted as to be scarcely recognizable in its true sense. A return to "fundamentalism" in extra-dimensional fantasy would no doubt pay dividends in more logical plots and greater realism. At the same time, the effort expended in producing literature in conformance with mathematical principles might uncover possibilities for science fiction which have scarcely been touched upon, and might eventually free it from its present shackles of materialism. Abstract geometry, by completely generalizing such concepts as "space" and "dimension", and by emphasizing the purely intellectual basis of such terms, constitutes a powerful tool for this purpose.

To the abstract mathematician, space is a completely undefined term in that he has no pre-conceived notions as to its properties. Instead, "space" is what his logic makes it; it is the entity which develops as a logical consequence of his original assumptions. These assumptions, be it understood, are in no way related to material reality, except possibly by coincidence; consequently the abstract mathematician accomplishes in one fell swoop what the science fictionist strives for in nibbles -- namely, complete freedom from his sense impressions.

This does not mean that the assumptions of the mathematician are more spectacular than those of the fiction writer. In fact, the reverse is true, for the whole super-structure of both Euclidean and non-Euclidean geometry in any number of dimensions may be developed from assumptions which it would be more unreasonable to refuse than to accept.

When the mathematician comes up with something which he calls the fourth dimension, therefore, he does not attempt to "prove" its existence by means of materialistic analogies, because he does not need to. In fact, he would probably be inclined to say that the "fourth dimension" certainly does not exist, if by "existence" is meant its identification with certain material attributes. On the other hand, if its existence is interpreted as its intellectual contemplation, which after all is perhaps the only true definition of existence, then the fourth dimension, and all the other fairyland entities of mathematics, is a foregone conclusion. Higher dimensions exist conceptually, not perceptually.

The abstract mathematician deals with completely undefined terms and then develops the relationships among those elements. He begins, for example, by assuming that there exists at least <u>one</u> such element, called a <u>point</u>, certainly not an unreasonable assumption. If one such point exists, then there may or may not be another such point. If he assumes the latter, then his logical development dies stillborn for want of courage; therefore, for the sake of mental exercise, if nothing else, he prefers to assume the former. By so doing, he provides himself with the minimum number of elements necessary to define a <u>relationship</u>; this relationship between the two elements p and q he calls

distance, which he defines as a one-dimensional relationship because it can be characterized by a single measurement. How this measurement is made, however, is purely arbitrary, and defining it will define the entire "metric" of the "space" which will eventually evolve.

In Euclidean one-dimensional space the distance pq has the property that

$$pq = p - q$$

but it is to be understood that by changing this property appropriately we may define the metrics of various non-Euclidean spaces, such as Riemannian space, Lobachevskian space, Hilbert space, or what have you. Again, whether the given definition conforms to our notion of the Lebensraum, the "life space", is a matter of no concern.

Let us now investigate the one-dimensional "space" which the abstract mathematician creates by way of his simple assumptions regarding two points and the distance relationship between them. If we consider a third point r, and its distances pr and qr from p and q respectively, we see that only one of these distances may be assigned arbitrarily, for this having been done, the other distance is automatically determined also. This restriction upon the distances between all possible pairs of points is expressed algebraically by the relationship

$$pq + qr = pr$$

and is a characteristic feature of our one-dimensional Euclidean metric. In fact, if this condition is not satisfied, then the third point does not lie in the one-dimensional "space" as defined and we must investigate its relationship to the other two points by enlarging our concept of "space" in such a way as to include a "second" dimension.

If we lack the intellectual curiosity to take this "bold" step, as we inevitable will when we come to the "fourth" dimension, we need only reflect that "distances" involve numbers, and that there is a veritable infinitude of triples of numbers a, b, c such that $a + b$ does not equal c. For example, $3 + 4 \neq 5$; therefore, why should it not be possible for three d points to be so situated that their respective distances by pairs are as 3:4:5? The answer is obvious.

The implication of the above reasoning is that the concept of "dimension" is intimately associated with the concept of "number", for inherent relationships among numbers have forced us to postulate a second dimension. Thus we reject the relationship

$$pq + qr = pr$$

which certainly does not account for all possible triples of numbers in favor of the more general and more inclusive relationship

$$pq + qr \geq pr.$$

This expression, which constitutes the familiar requirement that the sum of two sides of a triangle exceed the third, is essential to all metric spaces. In contrast to the one-dimensional requirement, three distances may satisfy this latter relationship without any one distance being uniquely determined by the other two. For example, if $pq = 3$ and $qr = 4$, then pr may assume any value between 1 and 7,

whereas in 1-dimensional space it would have been uniquely determined at the latter value (i.e. 7). The structure of this space is therefore considerably different from the structure of one-dimensional space, and the mathematician attributes its greater degree of freedom to what he calls, for lack of a better term, the second dimension.

That the theory of numbers forces us to a consideration of spaces of 3, 4, ... n dimensions may be readily demonstrated by pursuing the above considerations for 4, 5, ... n + 1 points, and the 6, 10, ... $\frac{1}{2}$ n (n + 1) distances determined by them. For example, when we consider the six distances determined by four points in the same plane, we observe that only five of these distances may be determined arbitrarily, for this having been done, the sixth is uniquely determined also. This is a consequence of the Heron formula for the area of a triangle in terms of the length of its three sides.

There exists, however, an infinity of groups of six numbers which do not fulfill the requirement laid down by the Heron formula. Consequently, it should be possible to locate four points in such a way that each of the six distances may be varied independently of the other five. This may be accomplished, of course, by introducing a third dimension, and by arranging the four points as the vertices of a tetrahedron.

As we might well expect, however, the dilemma reappears when we introduce the fifth point, for the ten consequent distances between pairs of points must satisfy a certain "five-point" condition. This condition is expressed by the determinant shown on the cover page.

As before, however, we can show that there exists an infinitude of aggregates of ten numbers which certainly do no not satisfy this relation and again, as before, that this predicates an added dimension. Clearly we are at the gates of hyperspace, with key in hand, and with no choice but to enter. Unless we reject the whole concept of number, we can not refuse such entry, for to do so would be logically inconsistent. In other words, hyperspace is an inevitable consequence of our number system.

It may further be seen that our Euclidean metric is just as "abstract" as any other, inasmuch as it may be arrived at by reasoning from completely undefined elements. Therefore we have no real right to prefer this metric to any other -- e.g. the metric of Hilbert space - - except that it seems to conform best to our sense impressions. It has perceptual existence as well as conceptual existence. Perhaps, however, sense impressions themselves may be regarded as undefined elements. If so, "abstract psychology" may be as possible as abstract geometry, and the true nature of perception and apperception may evolve only as we understand the topological relationships of these concepts as strictly undefined terms.

Attempts to "prove" the existence of hyperspace have been as numerous as they have been sterile. Symmetric entities such as our two hands, we have been told, can be superimposed by rotation through a fourth dimension. Organic compounds exist in dextro- and laevo- rotatory forms which are mirror images of each other. Supernatural phenomena are possible by means of an extra dimension. All of these facts are interesting, but they do not "prove" the existence of higher dimensions in the sense that their authors intended. The

existence of higher dimensions is a conceptual one only, and much more rigorous proofs are available for this fact, as has been shown.

The miracles of fourth-dimension are performed daily, hourly, continually in our thought train. In four-dimensional space the contents of an egg can be consumed without breaking the shell and surgical operations can be performed without incisions. But is this any more miraculous than transporting ourselves, in our mind's eye, from a sordid desk in a stuffy office to a hole-in-one at the country club? When we realize that on such occasions we never bother with such trivia as the opening of doors, there is little doubt that our thought-world, our conceptual existence, is higher-dimensional.

Explanations and consequences of a theory of hyperspace form the warp and woof of extra-dimensional fantasy. It is a process by which the conceptual existence of higher dimensions are put into perceptual terms. Theories of hyperspace and of abstract spaces in general, however, are sufficiently fantastic in their strictly mathematical sense to preclude the necessity for decoration. At the same time they possess that logical rigor which is often so consistently lacking in extra-dimensional fantasy.

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BOOK REVIEWS

The Educa tion of T. C. Mits, by L.R. Lieber, illus. by H. G. Lieber. Published by W.W. Norton & Company.

This book is for someone who is just beginning to be interested in abstract mathematics. However, the "old-timer" will probably get a kick out of it, too.

The book is bound in red and printed on good paper. It appears to be written in free verse, but the author states that:

"This is not intended to be
free verse.
Writing each phrase on a separate line
facilitates rapid reading,
and everyone
is in a hurry
nowadays."

T C. Mits is The Celebrated Man In The Street, meaning the reader. You are taken on a short tour of the land of mathematics. The author touches on Analytic Geometry, Calculus, Non-Euclidean Geometry, the Theory of Relativity, and more. There is also an interesting thing called "Our Totem Pole." It has five floors, and any invention can be traced through all five (or so the author says). On the first floor are the present-day inventions and the people who make them: mass-production and that sort of thing. On the second floor, there are the technicians who try to improve on the gadgets. The third floor brings the inventors themselves, who act on the laws discovered by fourth-floor men, the "pure" scientists. Of course, the scientists would have a hard time without the help of the "pure" mathematicians (top floor). I'd like to see the mouse-trap traced to the fifth floor.

-- Arnim Seielstad

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